Adaptive Level Control – Appendix A
Simple Generic Form of Equations for Identification of Process Dynamics

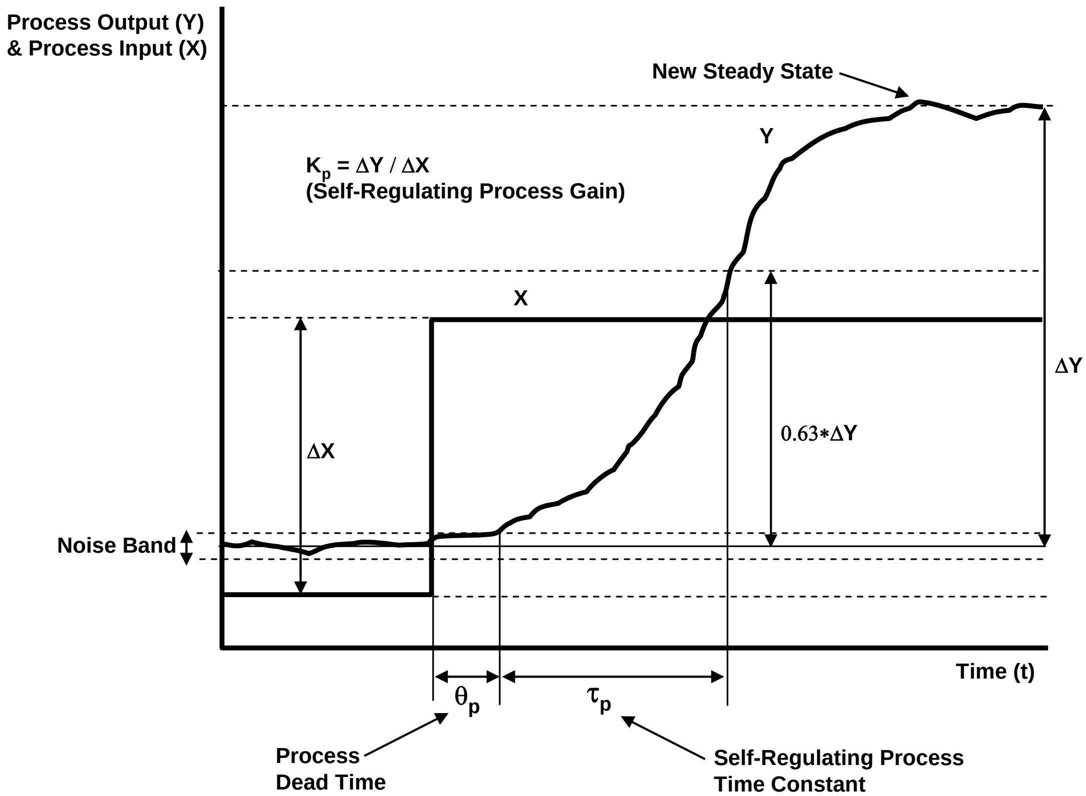
The process gains and time constants can be readily identified if the ordinary differential equations (*ODE*) for the rate of accumulation of energy or material in the volume are set up so that the process output of interest (*Y*) is on the right side of the equation with a unity coefficient. From this simple generic form we can identify the process time constant (τ_p) as the coefficient of derivative of the process output (*dY/dt*) and the process gain (K_p) as the coefficient of the process input (*X*). The process output (*Y*) and input (*X*) can be viewed as the controlled and manipulated variables, respectively. Many other terms can exist but these are not shown in the following equations. If the extra terms cannot be lumped into the terms shown, the extra terms can be categorized as disturbances or loads.

If the sign of the unity coefficient of the process output on the right side is negative (Equation A-1), the process has negative feedback and is termed a "self-regulating process." As the process output changes, the negative feedback slows down and eventually halts the excursion of the process output at its new steady state when it balances out the effect of the process input and the disturbances.

$$\tau_p * dY / dt = K_p * X - Y \qquad \text{(A-1)}$$

The integration of Equation A-1 provides the decelerating exponential time response of a process with negative feedback per Equation A-2. The change in the process output (ΔY) for a change in the process input (ΔX) is:

$$\Delta Y = K_p * (1 - e^{-t/\tau_p}) * \Delta X \qquad \text{(A-2)}$$



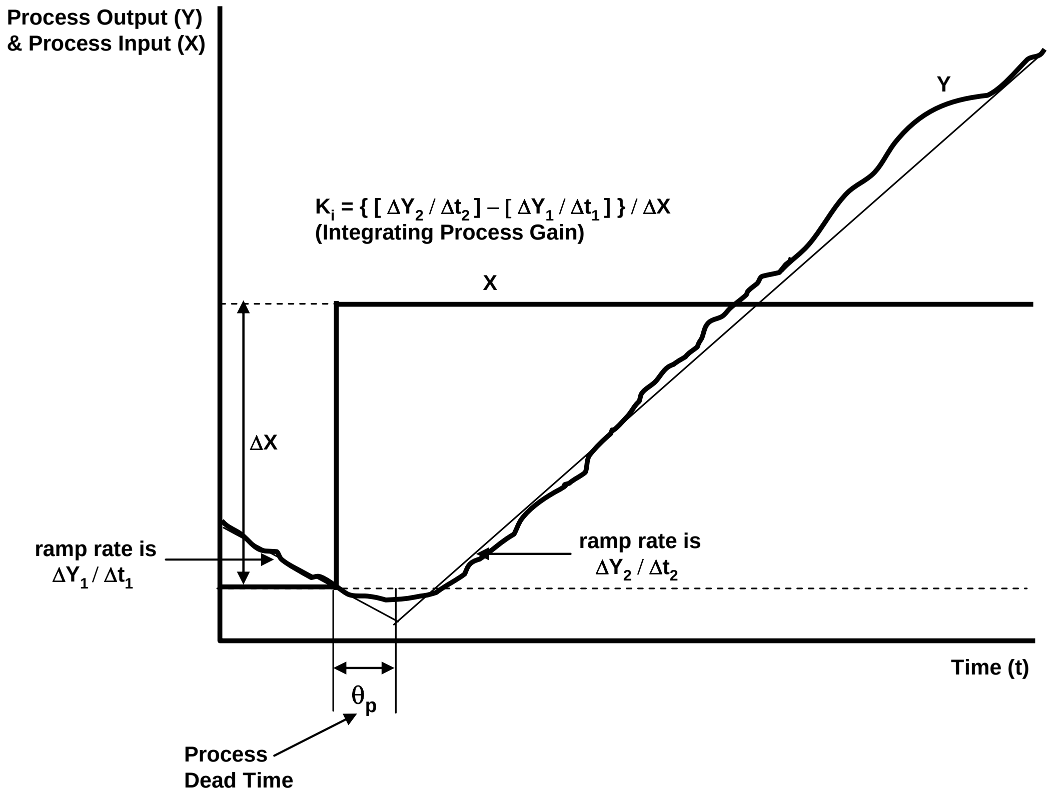
Open Loop Self-Regulating Response for Process with Negative Feedback

If the process output does not appear on the right side (Equation A-3), there is no process feedback and is termed an "integrating process." As the process output changes, there is no feedback to slow it down or speed it up so it continues to ramp. There is no steady state. The ramping will only stop when *X* is zero or balances out the disturbance or load.

$$dY / dt = K_i * X \qquad \text{(A-3)}$$

The integration of Equation A-3 provides the continual ramping time response of a process with no feedback per Equation A-4. The change in the process output (ΔY) for a change in the process input (ΔX) is:

$$\Delta Y = K_i * \Delta t * \Delta X \qquad \text{(A-4)}$$



Open Loop Integrating Response for Process with Zero Feedback

In the more important loops for control of large volumes, the time constant in Equation 1 is so often large that the time to reach steady state is beyond the time frame of interest. Since these loops with small dead-time-to-time constant ratios should be tuned with small lambda factors (high controller gains), the controller only sees the first part of the excursion before the inflection point and deceleration by negative process feedback. In this case we have a "Near Integrator," and Equation 1 is best visualized as Equation 3 with an integrator gain calculated per Equation A-5

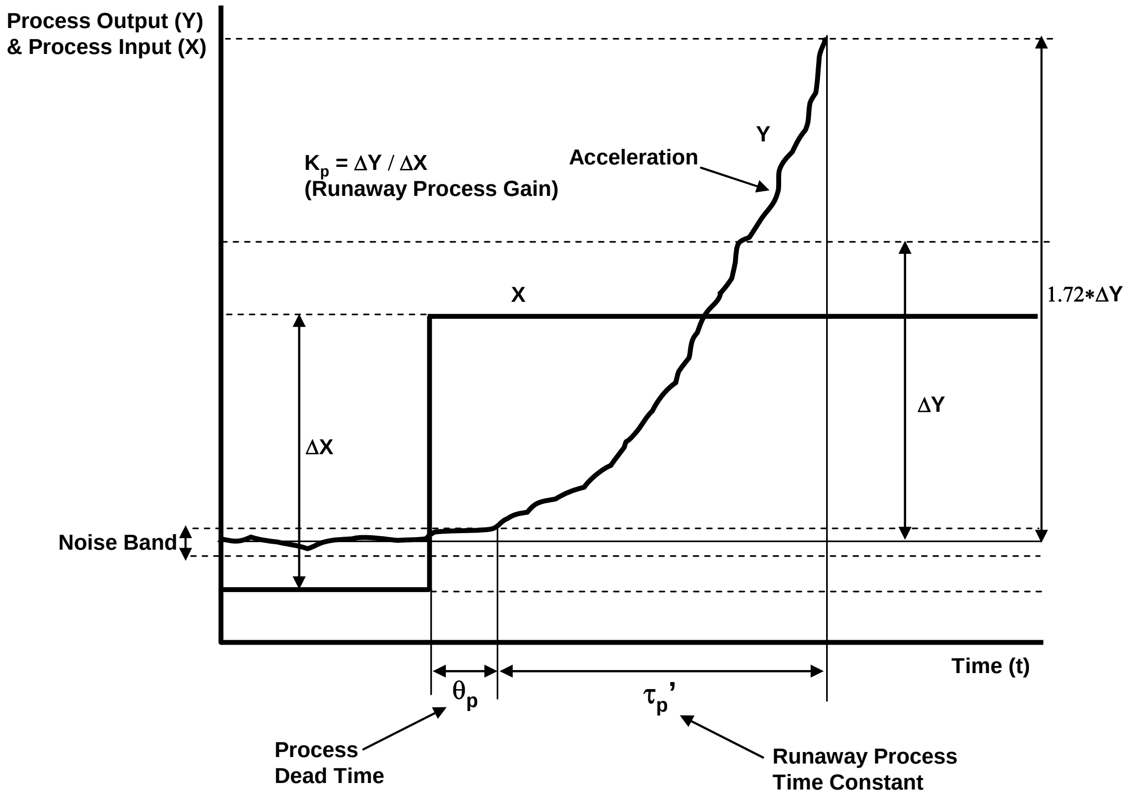
$$K_i = K_p / \tau_p \qquad \text{(A-5)}$$

If the sign of the unity coefficient of the process output on the right side is positive (Equation A-6), the process has positive feedback and is termed a "runaway process." As the process output changes, the positive feedback speeds up the excursion unless disturbances counteract the effect of the process input and output.

$$\tau_p * dY / dt = K_p * X + Y \qquad \text{(A-6)}$$

The integration of Equation A-6 provides the accelerating exponential time response of positive feedback process per Equation A-7. The change in the process output (ΔY) for a change in the process input (ΔX) is:

$$\Delta Y = K_p * (e^{t/\tau_p} - 1) * \Delta X \qquad \text{(A-7)}$$



Open Loop Runaway Response for Process with Positive Feedback

General Level Dynamics for Vessel

The level response dynamics can be derived from a simple material balance where the rate of change of the mass in the volume is equal to the difference between the total flow that goes into and out of the vessel. If the liquid exiting flows are pumped and the changes in static head on pump flow rate are negligible, the exiting flows are independent of level. For this prevalent general case, the process output does not appear on the right side of the equation, and we have a response with no process feedback.

$$dM / dt = \sum F_i - \sum F_v - \Sigma F_x \qquad \text{(A-8)}$$

For a section of the volume, the mass can be expressed as the product of fluid density, vessel cross sectional area, and level.

$$d(\rho * A * L) / dt = \sum F_i - \sum F_v - \Sigma F_x \qquad \text{(A-9)}$$

If the changes in density and area are negligible during an integration step, we can focus the differential equation on the rate of change of level. We can divide through by product of density and area ($\rho_o * A_o$). Furthermore if an inlet flow is manipulated, the vapor and discharge flows with their process multiplier can be grouped as an outlet flow (F_o).

$$dL_o \; / \; dt = \left[1/(\rho * A)\right] * \left(F_i - F_o \right) \qquad \text{(A-10)}$$

Since the PID algorithm works with signals in percent, we need to convert the level to a process output (Y) in percent of maximum level and the manipulated flow to process input (X) in percent of maximum flow. The flows going out are expressed as a percent load (Z). For this example, we are manipulating the input flow. The maximums must be in consistent engineering units (e.g., meters for level and kg/sec for flow). The maximums are the measurement spans for level and flow ranges that start at zero.

$$\left(L_{\max} \; / \, 100\%\right) * dY \; / \; dt = \left[\left(F_{\max} \; / \, 100\%\right) / (\rho * A)\right] * (X \; - \; Z) \qquad \text{(A-11)}$$

If we divide through by the coefficient of the process output, the 100% cancels out making the equation simpler.

$$dY \; / \; dt = \left[\left(F_{\max} \; / \; L_{\max}\right) / (\rho * A)\right] * (X \; - \; Z) \qquad \text{(A-12)}$$

The coefficient of the manipulated process input is the integrating process gain. If we integrate the differential equation, we have a pure integrating response with an integrating gain that is the flow measurement span divided by the product of the maximum level.

$$K_i = F_{\max} \; / \; \left[(\rho * A) * L_{\max} \right] \qquad \text{(A-13)}$$

It is useful for control system analysis to detail the overall open loop gain for the process as a product of gains for the manipulated variable (K_{mv}), process variable (K_{pv}), and controlled variable (K_{cv}). The gains for manipulated and controlled variables are proportional and inversely proportional to the final control element flow capacity and the measurement span, respectively, in engineering units. The gain for the process variable is inversely proportional to the product of the density and cross sectional area. It is important that the product of these gains end up with an integrator gain for the process with inverse time units (e.g. 1/sec) by the cancellation out of engineering units.

$$K_i = K_{mv} * K_{pv} * K_{cv} \qquad \text{(A-14)}$$

$$K_{mv} = F_{\max} \; / \; 100\% \qquad \text{(A-15)}$$

$$K_{pv} = 1 \; / \; \left[(\rho * A) \right] \qquad \text{(A-16)}$$

$$K_{cv} = 100\% \; / \; L_{\max} \qquad \text{(A-17)}$$

Normally, level has an integrating response, but the dependence of the gravity discharge flow on the square root of the level makes this level response self-regulating.

The general form of the differential equation for a self-regulating process is:

$$\tau_p * dY/dt = K_p * X - Y \quad \text{(A-18)}$$

For the MIT Anna University Lab experiment with volumetric flow of water (specific gravity = 1.0) and using the equation for the volume of a cone, the manipulation of an input flow, and a gravity discharge flow through a valve on the vessel drain:

$$\left(\frac{\pi * r^2}{3}\right) * dh/dt = F_i - C * h^{1/2} \quad \text{(A-19)}$$

Since the PID algorithm uses percent signals, we convert the process output to a percent of level and the process input to a percent inlet flow.

$$\left(\frac{\pi * r^2}{3}\right) * \left(L_{\max}/100\%\right) * dY/dt = (F_{\max}/100\%) * X - C * h^{1/2} \quad \text{(A-20)}$$

To get it into the right form to identify the process gain and time constant, we multiply through by the square root of the level ($h^{1/2}$) and divide by the flow coefficient (C)

$$\left(\frac{\pi * r^2}{3 * C} * h^{1/2}\right) * dh/dt = \left[\left(\frac{h^{1/2}}{C}\right) * \left(F_{\max}/L_{\max}\right)\right] * X - h \quad \text{(A-21)}$$

From the above form of the differential equation, the time constant (τ_p) and the process gain (K_p) can be approximated for small changes in level as follows (the open loop gain can be split again into a product of manipulated, process and controlled variable gains, but here for a self-regulating process the gain is dimensionless):

$$\tau_p = \frac{\pi * r^2}{3 * C} * h^{1/2} \quad \text{(A-22)}$$

$$K_p = \frac{h^{1/2} * F_{\max}}{C * L_{\max}} \quad \text{(A-23)}$$

For large volumes, the level will appear to ramp in the control region. The "near integrating" process (K_i) gain for these conditions can be approximated as:

$$K_i = \frac{3 * F_{\max}}{\pi * r^2 * L_{\max}} \qquad \text{(A-24)}$$

Nomenclature

$A =$ cross-sectional area (squared meters)
$C =$ flow coefficient (MIT Anna Lab: cubic meters per sec water per square root meters)
$F_i =$ inlet flow to volume (kg/sec) (MIT Anna Lab: cubic meters per sec water)
$F_o =$ outlet flow from volume (kg/sec)
$F_v =$ vapor flow from volume (kg/sec)
$F_x =$ exit flow from volume (kg/sec)
$F_{\max} =$ maximum flow rate (kg/sec) (MIT Anna Lab: cubic meters per sec water)
$h =$ height of level in conical tank (meters)
$K_c =$ controller gain (dimensionless)
$K_i =$ integrating process gain (1/sec)
$K_p =$ self-regulating process gain (dimensionless)
$K_{cv} =$ controlled variable gain (% per meter for general case of vessel level)
$K_{mv} =$ manipulated variable gain (kg/sec per % for general case of vessel level)
$K_{pv} =$ process variable gain (meters per kg for general case of vessel level)
$L_{\max} =$ level measurement span (meters)
$r =$ radius of conical tank at surface (meters)
$\rho =$ density of fluid in vessel (kg per cubic meter)
$T_i =$ integral (reset) time (sec)
$\tau_p =$ process time constant (sec)
$\theta_p =$ process dead time (sec)
$X =$ process input (percent)
$Y =$ process output (percent)
$Z =$ process load (percent)