Appendix K

PID Forms and Conversion of Tuning Settings

Conversion of Settings to Correct Units and Form

Most Distributed Control Systems use the ISA Standard Form with specific tuning setting units where the controller gain is dimensionless, the reset time is seconds (seconds per repeat), and the rate time is seconds. Some controllers use a proportional band in percent rather than a gain. Proportional band is the percent change in PID error needed to cause a 100 percent change in PID output. Proportional-integral-derivative (PID) tuning settings must first be checked for units and if necessary undergo a units conversion before being used. After the settings with proper units are obtained, the settings must be converted from the assumed or previous existing Form to the present Form being used.

1. Convert proportional band (\%) to controller gain (%/\%) (dimensionless)
2. Gain = 100%/Proportional Band
3. Convert reset setting in repeats per minute to reset time in seconds per repeat
4. Seconds per repeat = 60/repeats per minute
5. Convert rate time in minutes to rate time in seconds
6. Seconds = 60 * minutes
7. After the tuning setting units are verified, convert Series and Parallel Forms tuning settings to and from the ISA Standard Form tuning settings per equations below. The primed tuning settings are for the Series Form, the double primed tuning settings are for the Parallel Form, and the unprimed tuning settings are for the ISA Standard Form used in this book.

To convert from Series to ISA Standard Form controller gain:

\[ K_c = \frac{T_r' + T_d'}{T_i'} * K_c' \]  \hspace{1cm} (K.1)

To convert from Series to ISA Standard Form reset (integral) time:

\[ T_i = \frac{T_i' + T_d'}{T_i'} * T_i' = T_i' + T_d' \]  \hspace{1cm} (K.2)

To convert from Series to ISA Standard Form rate time:

\[ T_d = \frac{T_i'}{T_i' + T_d'} * T_d' \]  \hspace{1cm} (K.3)

Note that if the rate time is zero, the ISA Standard Form (Figure K.1) and Series Form (Figure K.2) settings are identical. When using the ISA Standard Form, if the rate
time is greater than one-fourth the reset time the response can become oscillatory. If the rate time exceeds the reset time, the response can become unstable from a reversal of action form these modes. The Series Form inherently prevents this instability by increasing the effective reset time as the rate time is increased.

**Figure K.1.** ISA Standard Form has modes in parallel with proportional gain factor.

**Series Form in analog controllers and early DCS available as a choice in most modern DCS**

**Figure K.2.** Series Form has derivative mode computed in series with other modes.
We can convert from ISA Standard Form to the Series Form using the following equations if the reset time is equal to or greater than four times the rate time ($T_i \geq 4 * T_d$).

$$K'_c = \frac{K_c}{2} \left[ 1 + \left( 1 - 4 * \frac{T_d}{T_i} \right)^{0.5} \right]$$ (K.4)

$$T'_i = \frac{T_i}{2} \left[ 1 + \left( 1 - 4 * \frac{T_d}{T_i} \right)^{0.5} \right]$$ (K.5)

$$T'_d = \frac{T_i}{2} \left[ 1 - \left( 1 - 4 * \frac{T_d}{T_i} \right)^{0.5} \right]$$ (K.6)

Figure K.3. Parallel Form has independent mode gain settings.

The Parallel Form (Figure K.3) is depicted in many control theory textbooks but is rarely used in the process industries. The occasional use in some DCS and PLC to isolate the proportional mode tuning setting from the other modes is problematic in terms of creating dramatic differences in reset and rate settings causing significant confusion. The gain setting does not affect the contribution from the integral and derivative modes. Sometimes the integral and derivative mode settings are called integral gain and derivative gain, respectively.

We can convert from the Parallel Form to the ISA Standard Form using the following equations:
If the parallel form integral mode tuning parameter is an integral *time*:

\[ T_i^* = K_c^* \cdot T_i^* \]  \hspace{1cm} (K.8a)

If the parallel form integral mode tuning parameter is an integral *gain*:

\[ T_i^* = \frac{K_c^*}{K_i^*} \]  \hspace{1cm} (K.8b)

If the parallel form derivative mode tuning parameter is a derivative *time*:

\[ T_d^* = \frac{T_d^*}{K_c^*} \]  \hspace{1cm} (K.9a)

If the parallel form derivative mode tuning parameter is a derivative *gain*:

\[ T_d^* = \frac{K_d^*}{K_c^*} \]  \hspace{1cm} (K.9b)

We can convert from the ISA Standard Form to the Parallel Form by using the following equations:

\[ K_c^* = K_c \]  \hspace{1cm} (K.10)

If the parallel form integral mode tuning parameter is an integral *time*:

\[ T_i^* = \frac{T_i}{K_c} \]  \hspace{1cm} (K.11a)

If the parallel form integral mode tuning parameter is an integral *gain*:

\[ K_i^* = \frac{K_c}{T_i} \]  \hspace{1cm} (K.11b)

If the parallel form derivative mode tuning parameter is a derivative *time*:

\[ T_d^* = K_c \cdot T_d \]  \hspace{1cm} (K.12a)

If the parallel form derivative mode tuning parameter is a derivative *gain*:
\[ K_d^* = K_c \cdot T_d \] \hspace{1cm} (K.12b)

Note that the parallel form derivative time and derivative gain have the same value.

Where
\[
\begin{align*}
K_c &= \text{controller gain for ISA Standard Form (\%/\%)} \hspace{0.5cm} (\text{dimensionless}) \\
K_c' &= \text{controller gain for Series Form (\%/\%)} \hspace{0.5cm} (\text{dimensionless}) \\
K_c'' &= \text{controller gain for Parallel Form (\%/\%)} \hspace{0.5cm} (\text{dimensionless}) \\
K_i &= \text{integral gain for Parallel Form (1/seconds)} \\
K_d &= \text{derivative gain for Parallel Form (seconds)} \\
T_i &= \text{integral time (reset time) for ISA Standard Form (seconds)} \\
T_i' &= \text{integral time (reset time) for Series Form (seconds)} \\
T_i'' &= \text{integral time (reset time) for Parallel Form (seconds)} \\
T_d &= \text{derivative time (rate time) for ISA Standard Form (reset time) (seconds)} \\
T_d' &= \text{derivative time (rate time) for Series Form (seconds)} \\
T_d'' &= \text{derivative time (rate time) for Parallel Form (seconds)}
\end{align*}
\]

**Difference Equations for ISA Standard and Series Form**

The difference equations for a reverse acting ISA Standard Form PID with rate limiting are as follows where the filter time on the derivative mode is a fraction alpha (\(\alpha\)) of the rate time (alpha is PID parameter whose default is typically 0.125):

\[
\begin{align*}
%CO_n &= P_n + I_n + D_n + %CO_i \\
P_n &= K_c \cdot (\beta \cdot %SP_n - %PV_n) \\
I_n &= \frac{K_c}{T_i} \cdot (%SP_n - %PV_n) \cdot \Delta t_x + I_{n-1} \\
D_n &= \frac{K_c \cdot T_d \cdot \left[ \gamma \cdot (%SP_n - %SP_{n-1}) - (%PV_n - %PV_{n-1}) \right] + \alpha \cdot T_d \cdot D_{n-1}}{\alpha \cdot T_d + \Delta t_x}
\end{align*}
\]

The difference equations for a reverse acting Series Form PID with rate limiting are as follows where the filter time on the derivative mode is a fraction alpha (\(\alpha\)) of the rate time (alpha is PID parameter whose default is typically 0.125):

\[ %CO_n = PD_n' + ID_n' + %CO_i \] \hspace{1cm} (K.17)
\[ PD_n' = K_c' \ast (\beta \ast \%SP_n - \%PV_n - D_n') \]  \hspace{1cm} (K.18)

\[ ID_n' = \frac{K_c'}{T_i} \ast (\%SP_n - \%PV_n - D_n') \ast \Delta t_x + ID_{n-1}' \]  \hspace{1cm} (K.19)

\[ D_n' = \frac{K_c' \ast T_d' \ast \left[ y \ast (\%SP_n - \%SP_{n-1}) - (\%PV_n - \%PV_{n-1}) \right] + \alpha \ast T_{n} \ast D_{n-1}'}{\alpha \ast T_{d} + \Delta t_x} \]  \hspace{1cm} (K.20)

If the structure used does not have integral action, the term \%CO is an adjustable bias and integral term is zero (Equations K.15 and K.19) are not used.

**Positive Feedback Implementation of Integral Mode**

The positive feedback implementation of the integral mode as shown in Figure K.4 effectively yields the equations above as seen in the following derivation for a PI controller using Laplace transforms. Instead of an integrator, a filter whose input is the controller output and whose output is added to the contribution from the proportional mode in the positive feedback implementation. The filter time is the integral time setting. When external reset feedback is enabled, the input to the filter is switched from the controller output to the external reset feedback signal.

The use of the positive feedback implementation of the integral mode is the basis for the enhanced PID developed for wireless that has been shown to be so effective for analyzer besides wireless devices especially when the update time is much larger than the process 63 percent response time.

\[ O(s) = K_c \ast E(s) + \frac{1}{1 + T_i \ast s} \ast O(s) \]  \hspace{1cm} (K.21)

\[ O(s) \ast \left( 1 - \frac{1}{1 + T_i \ast s} \right) = K_c \ast E(s) \]  \hspace{1cm} (K.22)

\[ O(s) \ast \left( \frac{T_i \ast s}{1 + T_i \ast s} \right) = K_c \ast E(s) \]  \hspace{1cm} (K.23)

\[ \frac{O(s)}{E(s)} = K_c \ast \left( \frac{1 + T_i \ast s}{T_i \ast s} \right) = K_c + \frac{K_c}{T_i \ast s} \]  \hspace{1cm} (K.24)

Where

\%CO = controller output at the transition from MAN or ROUT modes (%)

Finally Figure K.4 attempts to provide the block diagram for the ISA Standard Form with setpoint weight factors and the positive feedback implementation of integral action, which provides the considerable advantage of external reset feedback (e.g., dynamic reset limiting). The positive feedback implementation is the key to the enhanced PID found to be beneficial for control loops that use wireless devices and at-line or offline analyzers, have slow secondary loops or valves with excessive backlash or resolution limits or need
directional move suppression. Showing the positive feedback implementation in the time domain is no easy task since what is in the literature is the simple Laplace representation for the Series Form without setpoint weight factors and without feedforward control.

**Figure K.4.** ISA Standard Form with positive feedback implementation of integral mode and external reset feedback.