Appendix K – PID Controller Forms and Structures

Conversion of Settings to Correct Units and Form

The equations in this book are based on the ISA Standard Form with specific tuning setting units where the controller gain is dimensionless, the reset time is seconds (seconds per repeat), and the rate time is seconds. PID tuning settings must first be checked for units and if necessary undergo a units conversion before being used.

(1) Convert proportional band (%) to controller gain (%/%) (dimensionless)

\[ \text{Gain} = \frac{100 \%}{\text{Proportional Band}} \]

(2) Convert reset setting in repeats per minute to reset time in seconds per repeat

Seconds per repeat = \( \frac{60}{ \text{repeats per minute}} \)

(3) Convert rate time in minutes to rate time in seconds

Seconds = \( 60 \times \text{minutes} \)

(4) After the tuning setting units are verified, Series and Parallel Forms convert tuning settings to the ISA Standard Form tuning settings per equations below. The primed tuning settings are for the Series Form, the double primed tuning settings are for the Parallel Form, and the unprimed tuning settings are for the ISA Standard Form used in this book.

To convert from Series to ISA Standard Form controller gain:

\[ K_c = \frac{T'_i + T'_d}{T'_i} \times K'c \]  \hspace{1cm} (K1)

To convert from Series to ISA Standard Form reset (integral) time:

\[ T'_i = \frac{T'_i + T'_d}{T'_i} \times T'_i = T'_i + T'_d \]  \hspace{1cm} (K2)

To convert from Series to ISA Standard Form rate time:

\[ T'_d = \frac{T'_i}{T'_i + T'_d} \times T'_d \]  \hspace{1cm} (K3)
Note that if the rate time is zero, the ISA Standard and Series Form settings are identical. When using the ISA Standard Form, if the rate time is greater than \( \frac{1}{4} \) the reset time the response can become oscillatory. If the rate time exceeds the reset time, the response can become unstable from a reversal of action form these modes. The Series Form inherently prevents this instability by increasing the effective reset time as the rate time is increased.

We can convert from ISA Standard Form to the Series Form using the following equations if the reset time is equal to or greater than 4 times the rate time \( (T_i \geq 4 * T_d) \).

\[
K'_c = \frac{K_c}{2} \left[ 1 + \left( 1 - 4 \frac{T_d}{T_i} \right)^{0.5} \right]
\]  
(K4)

\[
T'_i = \frac{T_i}{2} \left[ 1 + \left( 1 - 4 \frac{T_d}{T_i} \right)^{0.5} \right]
\]  
(K5)

\[
T'_d = \frac{T_i}{2} \left[ 1 - \left( 1 - 4 \frac{T_d}{T_i} \right)^{0.5} \right]
\]  
(K6)

The Parallel form is used in some DCS and PLC to isolate the proportional mode tuning setting from the other the modes. The gain setting does not affect the contribution from the integral and derivative modes and is sometimes called non-interacting. We can convert from the Parallel to the ISA Standard Form using the following equations.

\[
K_c = K''_c
\]  
(K7)

\[
T_i = K''_c * T''_i
\]  
(K8)

\[
T_d = \frac{T''_d}{K_c}
\]  
(K9)

We can convert from the ISA Standard Form to the Parallel Form by using the following equations:

\[
K''_c = K_c
\]  
(K10)

\[
T''_i = \frac{T_i}{K_c}
\]  
(K11)
\[ T_d^* = K_c \cdot T_d \] (K12)

Where:

- \( K_c \) = controller gain for ISA Standard Form (%/%) (dimensionless)
- \( K_c' \) = controller gain for Series Form (%/%) (dimensionless)
- \( K_c'' \) = controller gain for Parallel Form (%/%) (dimensionless)
- \( T_i \) = integral time (reset time) for ISA Standard Form (seconds)
- \( T_i' \) = integral time (reset time) for Series Form (seconds)
- \( T_i'' \) = integral time (reset time) for Parallel Form (seconds)
- \( T_i' \) = derivative time (rate time) for ISA Standard Form (reset time) (seconds)
- \( T_i'' \) = integral time (rate time) for Series Form (seconds)
- \( T_i''' \) = integral time (rate time) for Parallel Form (seconds)

**Difference Equations for ISA Standard and Series Form**

The difference equations for a reverse acting ISA Standard Form PID with rate limiting where the filter time on the derivative mode is a fraction \( \alpha \) of the rate time is as follows:

\[ \%CO_n = P_n + I_n + D_n + \%CO_i \] (K13)

\[ P_n = K_c \cdot (\beta \cdot \%SP_n - \%PV_n) \] (K14)

\[ I_n = \frac{K_c}{T_i} \cdot (\%SP_n - \%PV_n) \cdot \Delta t_s + I_{n-1} \] (K15)

\[ D_n = \frac{K_c \cdot T_d \cdot \left[ y \cdot (\%SP_n - \%SP_{n-1}) - (\%PV_n - \%PV_{n-1}) \right] + \alpha \cdot T_d \cdot D_{n-1}}{\alpha \cdot T_d + \Delta t_s} \] (K16)

The difference equations for a reverse acting Series Form PID with rate limiting where the filter time on the derivative mode is a fraction \( \alpha \) of the rate time is as follows:

\[ \%CO_n = PD_n' + ID_n' + \%CO_i \] (K17)

\[ PD_n' = K_c' \cdot (\beta \cdot \%SP_n - \%PV_n - D_n') \] (K18)
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\[
ID'_n = \frac{K'_c}{T'_i} * (\%SP_n - \%PV_n - D'_n) \Delta t_x + ID'_{n-1} \tag{K19}
\]

\[
D'_n = \frac{K'_c T'_d}{\alpha} \left[ \nu \left( \%SP_n - \%SP_{n-1} \right) - \left( \%PV_n - \%PV_{n-1} \right) \right] + \frac{\alpha \cdot T'_n \cdot D'_{n-1}}{\alpha \cdot T'_d + \Delta t_x} \tag{K20}
\]

If the structure used does not have integral action, the term \%CO_i is an adjustable bias and integral term is zero (equations K-15 and K-19) are not used.

**Positive Feedback Implementation of Integral Mode**

The positive feedback implementation of the integral mode effectively yields the equations above as seen in the following derivation for a PI controller using Laplace transforms. Instead of an integrator, a filter whose input is the controller output and whose output is added to the contribution from the proportional mode in the positive feedback implementation. The filter time is the integral time setting. When external reset feedback is enabled, the input to the filter is switched from the controller output to the external reset feedback signal.

\[
O(s) = K_c \cdot E(s) + \frac{1}{1 + T_i \cdot s} \cdot O(s) \tag{K21}
\]

\[
O(s) \left( 1 - \frac{1}{1 + T_i \cdot s} \right) = K_c \cdot E(s) \tag{K22}
\]

\[
O(s) \cdot \frac{T_i \cdot s}{1 + T_i \cdot s} = K_c \cdot E(s) \tag{K23}
\]

\[
\frac{O(s)}{E(s)} = K_c \cdot \left( \frac{1 + T_i \cdot s}{T_i \cdot s} \right) = K_c + \frac{K_c}{T_i \cdot s} \tag{K24}
\]

Where:

\%CO_i = controller output at the transition from MAN or ROUT modes (%)
\%CO_n = controller output for current scan n (%)
\%CO_{n-1} = controller output for last scan n-1 (%)
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\( %PV_n \) = process variable for current scan \( n \) (%)
\( %PV_{n-1} \) = process variable for last scan \( n-1 \) (%)
\( %SP_n \) = setpoint for current scan \( n \) (%)
\( %SP_{n-1} \) = setpoint for last scan \( n-1 \) (%)

\( K_c \) = controller gain for ISA Standard Form (%/%) (dimensionless)
\( K'_c \) = controller gain for Series Form (%/%) (dimensionless)
\( T_i \) = integral time (reset time) for ISA Standard Form (seconds)
\( T'_i \) = integral time (reset time) for Series Form (seconds)
\( T_d \) = derivative time (rate time) for ISA Standard Form (reset time) (seconds)
\( T'_d \) = derivative time (rate time) for Series Form (seconds)

\( P_n \) = proportional mode contribution for ISA Standard Form for current scan \( n \) (%)
\( P_{n-1} \) = proportional mode contribution for ISA Standard Form for last scan \( n-1 \) (%)
\( PD_n \) = proportional-derivative mode contribution for Series Form for current scan \( n \) (%)
\( PD_{n-1} \) = proportional-derivative mode contribution for Series Form for last scan \( n-1 \) (%)

\( I_n \) = proportional mode contribution for ISA Standard Form for current scan \( n \) (%)
\( I_{n-1} \) = proportional mode contribution for ISA Standard Form for last scan \( n-1 \) (%)
\( ID_n \) = proportional-derivative mode contribution for Series Form for current scan \( n \) (%)
\( ID_{n-1} \) = proportional-derivative mode contribution for Series Form for last scan \( n-1 \) (%)

\( \Delta t_e \) = execution time of PID block (seconds)
\( \alpha \) = alpha factor for derivative mode filter (fraction of rate time) (0-1) (dimensionless)
\( \beta \) = beta setpoint weight factor for proportional mode (0-1) (dimensionless)
\( \gamma \) = gamma setpoint weight factor for derivative mode (0-1) (dimensionless)

\( E(s) \) = Laplace transform of controller error (%)
\( O(s) \) = Laplace transform of controller output (%)